

## **Paper Review : Assortment, Shelf-Space Allocation.**

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### **Competitive Assortment**

- Anderson, Simon and Regis Renault, “Pricing, Product Diversity and Search Costs: A Bertrand-Chamberlin-Diamond Model” *RAND Journal of Economics*, Vol 30, No 4, Winter 1999.

Consumers search for lower prices and for products they like (match value). Although the former motivation has been widely studied in search models, the latter was not. Using a random utility approach an optimal stopping rule is found for search behavior. Assuming stopping rule demand function could be computed as a function of prices, and then equilibrium price is derived. The optimal number of firms and the effect of diversity on tastes are studied for this equilibrium. Additionally, it is shown that Diamond, Bertrand and Chamberlin paradoxes are limiting cases of the equilibrium here derived.

- Coughlan, Anne and Greg Shaaffer, “Price-Matching Guarantees, Retail Competition and Product Line Assortment” *Working Paper, The Bradley Policy Research Center, No FR03-29.*

This paper study stylized assortment decisions in the presence of price matching guarantees (PMG). A game theoretic approach is used in a duopoly where assortment decisions are made on maximum of 2 products. The game consists in three stages: (1) PMG/no PMG, (2) Assortment decisions and (3) prices decisions. This complete game is solved in three capacity cases: (a) no capacity constraints, (b) capacity constraints for only one player and (c) capacity constraints for both. The results show that the nature of the of product variety (substitutability) and capacity constraints are key factors that jointly determine the characteristics of the equilibrium.

- Cachon, Gerard, Christian Terwiesch and Yi Xu “The Impact of Consumer Search, Firm Entry and Competition on Assortment and Pricing” *Working Paper, The Wharton School, University of Pennsylvania, January 2006.*

This paper also considers matching and price motivations to engage in search behavior. Symmetric equilibrium is described assuming that each firm offers a unique set of products implicitly assuming costless recall. Both, sequential and parallel search behaviors are studied exhibiting the same qualitative behavior with respect the effect of entry and search cost in equilibrium assortment decisions. Finally, advertising effects are analyzed in the parallel search model.

- Zettelmeyer, Florian, "The Strategic Use of Consumer Search Cost" *Working Paper, Haas School of Business, University of California Berkeley*, September, 2000.

This article claims that firms may not want to make consumer search easy, even if it is costless for them to do so. The paper considers products for which increased knowledge about the product category generally decrease consumer's perceived differentiation (e.g. pain killers). The author proof that, assuming utilities are uniformly distributed in  $[0,1]$  and costless recall, there exists costs and utility correlation parameters values such that in the (subgame perfect) equilibrium, one firm charge higher prices but lower search costs.

- Watson, Randal "Product Variety and Competition in the Retail Market for Eyeglasses" *Working Paper, University Texas at Austin*, September, 2005.

Using an original data set, this paper studies empirically the effect of competition in assortment decisions in the eyeglasses industry. Assortment decision is the breath of the assortment measured as the number of products offered (horizontally differentiated goods). The model control for endogenous location decisions and use proximity of other stores as a measure of competitions. Results shows that although non linear reactions to changes in competition intensity. Facing a moderate increase in competition firms could increase assortment breath, but facing more competition, they will eventually start cutting product variety.

- Draganska, Michaela and Dipak Jain "Product Line Length as a Competitive Tool" *Journal of Economics and Management Strategy*, Vol 14, No 1, Spring 2005.

This article examines empirically how product line length affects manufacturer profitability in the yoghurt market. In the demand side, it is assumed that each consumer wants to buy a bundle of flavors and conditional on that, consumer select product line that is likely to offer the product they want. In the supply side, it is assumed that firms set prices and product-line length as a result of a Bertrand-Nash game. Results show that there are decreasing returns to product-line length and therefore exist and optimal length.

## **Assortment**

- Kahn Barbara and Donald Lehman. "Modeling Choice Among Assortments" *Journal of Retailing* Vol 67, No 3, Fall 1991.

The article proposes a model to describe choices among different assortment. If consumers have uncertainty about their futures preferences and therefore they value assortments high more flexibility in future choice. The model states is based in a bunch of postulates: (1) more acceptable options are better, more unacceptable options are worse (2) assortments including more preferred items are better (3) uniqueness of an item add value to an assortment. Two experimental studies are conducted to test the model and compare result to alternative models. The proposed model has the better fit, but hierarchical models could be a good alternative model.

- Van Ryzin Garret and Siddhart Mahajan. "On the Relationship Between Inventory Cost and Variety Benefits in Retail Assortments" *Marketing Science* Vol 45, No 11, Fall 1999.

This article develops a simplified model to determine optimal assortment considering both the demand side (preferences) and the supply side (inventory costs). Consumer's preferences are modeled using standard MNL model while the inventory is modeled using the newsboy model. Combining these two models, a theoretical description of the optimal assortment is presented. Among the underlying assumptions in the model is the identical prices and costs for all alternatives.

- Siddhart Mahajan and Garret van Ryzin "Stocking Retail Assortment Under Dynamic Consumer Substitution" *Operations Research* Vol 49, No 3, May-June 2001.

This article focuses on the inventory decisions where consumer can decide to buy another product when their favorite one is out of stock. A characterization of the demand and profit function is developed showing that profit function is not well behaved. In particular sales are not component-wise concave in the inventory levels making harder the inventory problem. A continuous approximation and a gradient-based algorithm to solve is then developed. The model has a major limitation. The model uses a newsvendor approach to describe inventory decisions, where retailer have to decide only once inventory levels for each category, could be suitable for fashion or very seasonal categories, but not for groceries.

- Hoch, Stephen, Eric Bradlow and Brian Wansink. "The Variety of an Assortment" *Marketing Science* Vol 18, No 4, 1999.

This paper presents a formal mathematical model to describe the perceived variety in a given assortment considering both, the structure and the spatial location. Structure is modeled using pair wise measure of similarity of attributes and spatial location is modeled allowing different weight to pair wise comparison of adjacent and no adjacent products. The number of pair wise comparisons is so high than some behavioral constraints are added. The model is tested in a laboratory study. The major findings are: (1) Structure has a big impact on variety. (2) Proximity Matters (3) Organized display is preferred in analytical processing while Randomized display is preferred in holistic evaluations (4) People prefer organized assortment in store choice tasks.

- van Herpen, Erica and Rick Pieters. "The Variety of an Assortment: An Extension to the Attribute-Based Approach" Research note, *Marketing Science* Vol 21, No 3, Summer 2002.

In Hoch, Bradlow and Wasnick paper (1998, hereafter HBW), a product-based approach is used to describe the variety of an assortment. This article claims that a attribute-based approach could be more adequate. The product-based measures use the dissimilarity of products in an assortment across attributes. The attribute-based measures use the dispersion of products across products (Entropy) and the dissociation between product attributes (1-

Lambda). The major findings are that attribute-based approach (1) is less sensible to size of the assortment (2) predicts better variety perception.

- Bradlow, Eric and Vithala Rao. "A Hierarchical Bayes Model for Assortment Choice" *Journal of Marketing Research* Vol 37, May 2000.

Traditional discrete choice models focus on decisions where people chose a single alternative in a fixed set. This paper focuses on decision where people can choose assortment of alternatives. A hierarchical Bayes model is developed based on a balance model (Farquhar and Rao, 1976) where the utility of an assortment is given by a weighted sum of means and dispersions. A normal distribution is assumed for the parameters and the means of these normal distributions. The model is calibrated using laboratory data. The approach is useful to answer questions such as which assortment of a given size has the higher number of customer who prefer it over any other? For a given assortment, what type of customers is likely to purchase it?, etc.

- Chong, Juin-Kuan, Teck-Hua Ho and Christopher Tang "A Modeling Framework for Category Assortment Planning" *Manufacturing & Service Operation Management* Vol 3, No 3, Summer 2001.

The content of this article can be divided into two main parts. First, an extension of Guadagni and Little model (1983) is developed adding three explanatory variables in the indirect utility functions. These new variables try to describe the variety are based on the three structure of the products included in the assortment. They are the attraction of a brand to a consumer during a trip (based on previous purchases of products covered by the brand) and the disappointment of a brand to a consumer during a trip (based on previous purchases of products not covered by the brand). The estimation procedures show a significant effect of these covariates in several categories. The second part of the paper use this model combined with a purchase incidence model to evaluate the impact of different assortment in profitability. A heuristic approach is used to find close to optimal assortment configurations.

- Boatwright, Peter and Joseph Nunes "Reducing Assortment: An Attribute-Based Approach" *Journal Of Marketing* Vol 65, July 2001.

A natural experiment where an online retail reduced the number of items offered in several categories to a set of its panelists allows studying important characteristics of the impact of this reduction. In particular, how different types of SKU reductions (in terms of availability of attributes) affect purchase behavior at individual level. A customized discrete choice model is used with market share, number of items (SKUs), number of brands, number of sizes, number of brand-sizes, number of flavors and category price as covariates and a random coefficient structure. The results show that (1) consumer experienced divergent reactions to reductions (2) sales increased, but likelihood of making a purchase decreased and (3) changes in availability of attributes affect consumers differently.

- Caro, Felipe and Jeremy Gallien "Dynamic Assortment with Demand Learning for Seasonal Consumer Good" *Working Paper*, November 2005.

This paper studies the assortment problem for fast fashion retailers with quick response supply chains. Here, the company can dynamically modify the assortment based on the observed sales of the product in the season. A modification of multiarmed bandit model is proposed to take into account lead times, new product costs and substitution effects. A closed form solution is proposed derived from duality theory and several approximations. Numerical results show good performance with the advantage of easiness to implement and interpret.

### **Shelf-Space Allocation**

- Corstjen, Marcel and Peter Doyle "A Model for Optimizing Retail Space Allocations" *Management Science* Vol 27, No 7, July 1981.

This is the first paper describe the first model to decide shelf-space allocation that incorporates the four key determinants of shelf-space store profitability: product space elasticity, inter-product cross elasticities, different product profit margins and handling and inventory costs. A geometric programming algorithm is used to solve small instances of the problem.

- Zufryden, Fred "A Dynamic Programming Approach for Product Selection and Supermarket Shelf-Space Allocation" *Journal of Operational Research Society* Vol 37, No 4, 1986.

This paper generalizes Corstjen and Doyle paper (1981) allowing much more general forms for the demand functions. A dynamic programming approach is used to solve the problem finding optimal solutions to real-scale problems in a personal computer.

- Bultez, Alain and Philippe Naert "SH.A.R.P: Shelf Allocation for Retailers Profit" *Marketing Science* Vol 7, No 3, Summer 1988.

SH.A.R.P is an optimization model to allocate shelf space to a set of alternatives in a category in the Retail industry. The objective function is the maximization of the total profit (gross margin x sales – replenishment costs) subject to a unique space availability constraint. Defining sales as a flexible multivariate logistic function of space allocation, an analytical condition is found imposing first order conditions. Then, a heuristic method is developed to find the near to optimal solution that have been implemented in several supermarket and categories with promising results. The major contribution of this paper is it constitutes the first model that incorporates product interdependencies within product class in the demand estimation beyond the space constraint.

- Borin, Norm, Paul Farris and James Freeland "A Model for Determining Retail Product Category Assortment and Shelf Space Allocation" *Decision Sciences* Vol 25, No 3, May/June 1994.

A mathematical programming model is presented where the objective function is the return over inventory. The demand function considers intrinsic effect, in store marketing variables, stockouts and assortment effects. The problem is solved using a simulated annealing heuristic.

- Irion, Jens, Faiz Al-Khayyal and Jye-Chyi Lu "A Piecewise Linearization Framework for Retail Shelf Space Management Models" *Working Paper*, July 2004.

Based on a Corstjen and Doyle paper (1981) a mathematical programming model is presented considering both the demand and cost side. As the resulting problem is nonlinear integer problem it cannot be solved for instances of real size. Then a linearization of the objective function is made. After linearization, the problem can be solved using standard optimization packages.

- Cachon, Gerard, Christian Terwiesch and Yi Xu, "Retail Assortment Planning in the Presence of Consumer Search" *Manufacturing & Service Operations Management*, Vol 7, No 4, Fall 2005.

The main contribution is proving that ignoring search in assortment planning model is likely to lead to assortment that are too narrow because doing so removes a key benefit from expanding an assortment, the benefit of preventing consumer search. Three analytical models are developed to describe purchase behavior. A standard MNL model, a *independent* model where consumer expect to observe different assortment by searching and an *overlapping* one where deeper assortment reducing the value of searching. The main drawback of considering value depending on assortment is the difficulty on demand parameter estimation. Numerical results show some robustness in the estimation procedure. Three important critiques can be raised. First, the model assume a there is no price difference across retailers, which is not a suitable assumption for a majority of applied cases. Second, heterogeneity in consumer preferences is not considered. Finally, there is no shelf constraint in the assortment planning.

- Cachon, Gerard and Gurhan Kok "Category Management and Coordination in Retail Assortment in the Presence of Basket Shopping Consumers" *Working Paper*, July 2004.

This article analyze the optimal assortment decision assuming that consumer buy in multiple categories and therefore they made their store decision based on the complete set of assortment of all categories in their baskets. A comparison between a centralized and category management assortment decision is made concluding that a decentralized decision process could imply inefficient decision. However, a decentralized decision could be close to optimal if an appropriate metric of evaluation. Although the game theoretic approach used is interesting, it is not very realistic to assume that category managers made assortment decisions in a strategic way and based on assortment decisions of other categories.

- Kok, Gurhan and Marshall Fischer "Demand Estimation and Assortment Optimization under Substitution: Methodology and Application" *Working Paper*, January 2006.

The assortment problem is modeled as the maximization of the total revenues in the category subject to a unique space constraint. Three different procedures to estimate demand parameters are presented depending on the data availability. In all these models it is assumed that consumers may buy substitutes when their favorite product is unavailable. Then a heuristic procedure to find an optimal solution is described and formally discussed. Finally an empirical description is done based on the heuristic solutions for several cases. The solution presented has two limitations. First, the set of alternative SKUs that can be included in the assortment is given by the products that are already sold by at least one store in the chain limiting the flexibility of the product selection. Second, the parameter estimation is based on the assumption that there is a single substitution rate for the whole category. Although some of the estimation approaches presented allow for more flexible forms, the results are not fully discussed.