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# Retail Assortment Planning: Demand Estimation and Optimization Approaches

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CERET 2011

# Outline

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- Industry Approach

- Best Buy
- Borders
- Albert Heijn (Ahold)
- Tanishq, India

(Kok and Fisher and Vaidyanathan, 2008, Book chapter)

- Academic Approach

- Kok and Fisher, 2007, *Operations Research*
- Fisher and Vaidyanathan, 2007, Working paper
- Kok and Xu, 2011, *Management Science*
- Bernstein, Kok, and Xie, 2010, Working paper

# Definition of an Assortment

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- Category examples
  - Men's dress shirts, Sunglasses, DVD players, Cereals
- Assortment at category level
  - Breadth: Number of categories
  - Depth: “size of assortment” in each category
- Assortment at product level
  - A selection of product variants within a category
  - Products are differentiated by some attributes
  - Products are potential substitutes
  - Number of SKUs or facings is limited by category shelf space

# Assortment Planning at Best Buy

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- Hierarchical planning
  - Focus on buying at chain level
  - Deployment to stores (based on store clusters, climate zone etc.)
- Dynamic environment
  - Identifying growth opportunities (digital vs. traditional camera)
  - New products
  - Short life cycle (Newsvendor kind of addresses that)
  - Markdown pricing
- Not every category is the same
  - Identifying critical resources (Promo, labor, impulse, price, selection) for different categories

# Critical to Align Assortment with Company Strategy...

## Primary Attribute

Secondary Attribute

	Price	Service	Product	Experience	Access
Price					
Service					
Product					
Experience					
Access					

## ...and Allocate Resources based on Marginal Returns

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Category	Promo	Labor	Impulse	Price	Selection
Computer	High	High	Low	High	Medium
Refrigerator	Medium	High	Low	Medium	High
Accessories	Low	Low	High	Low	Low
Movies	High	Med	High	High	High

# Identify Growth Opportunities

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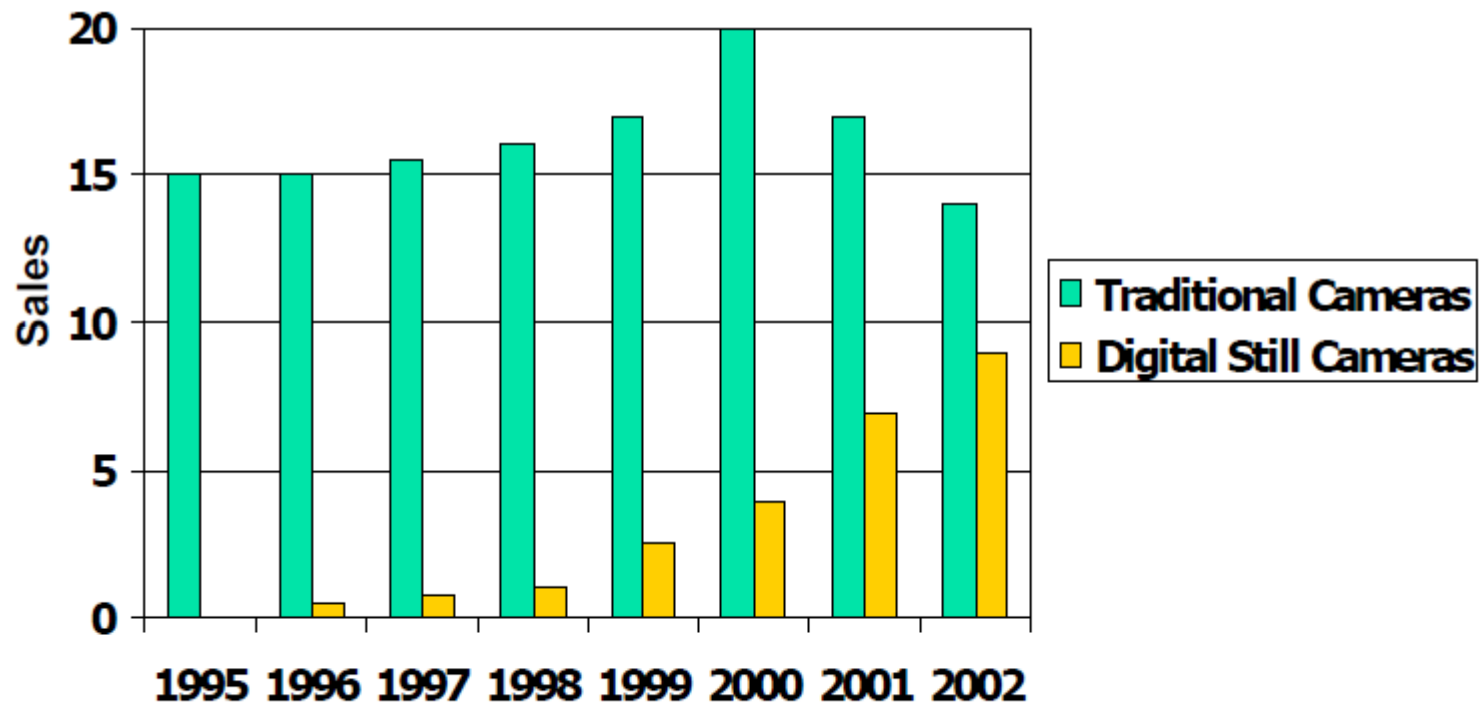
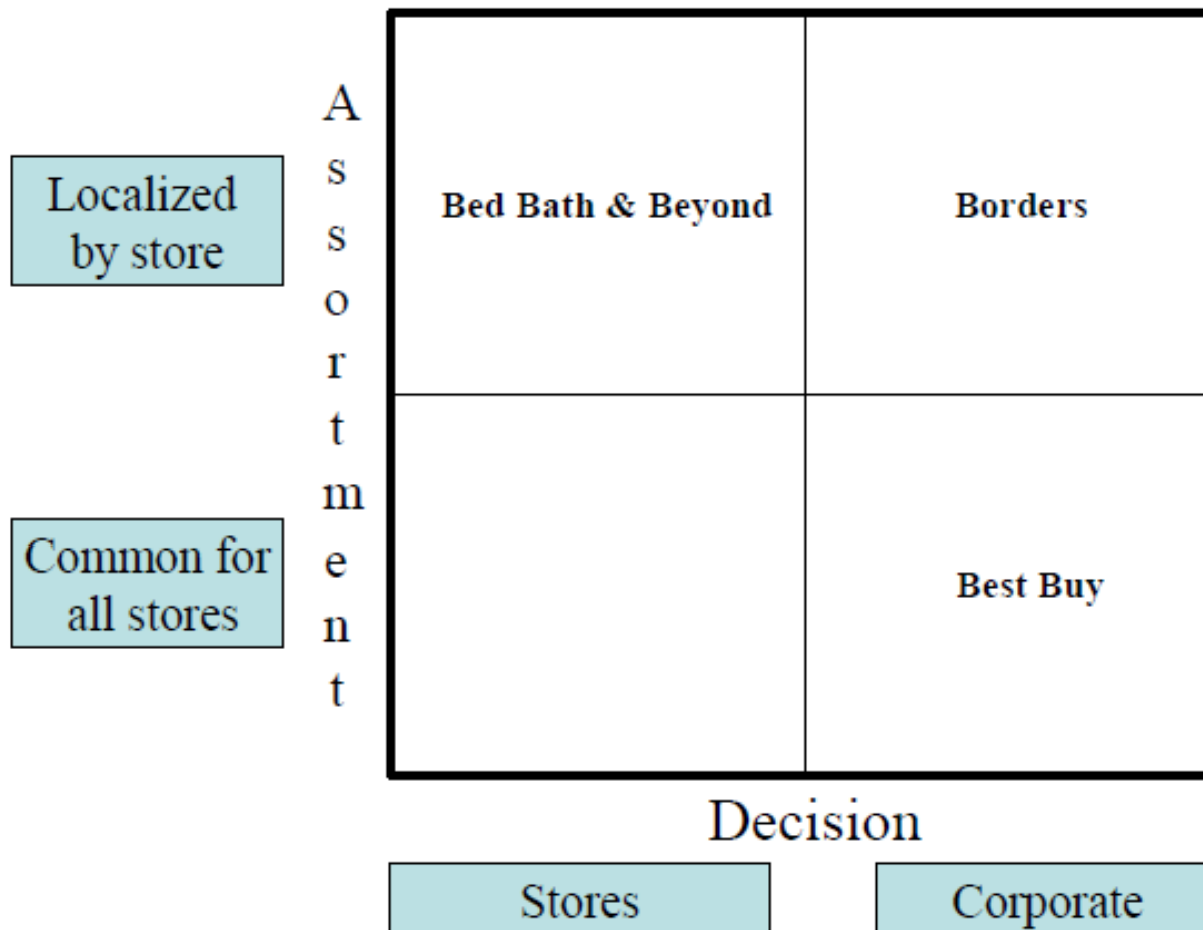


Figure 2: Historical sales of traditional and digital cameras

# Assortment Localization

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# Assortment Planning at Borders

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- Basic Premise – “Except for best sellers, customer is interested NOT in title BUT category”
- 300,000 titles grouped into 300 categories
- Category popularity assessed by computing Relative Sales Per Title

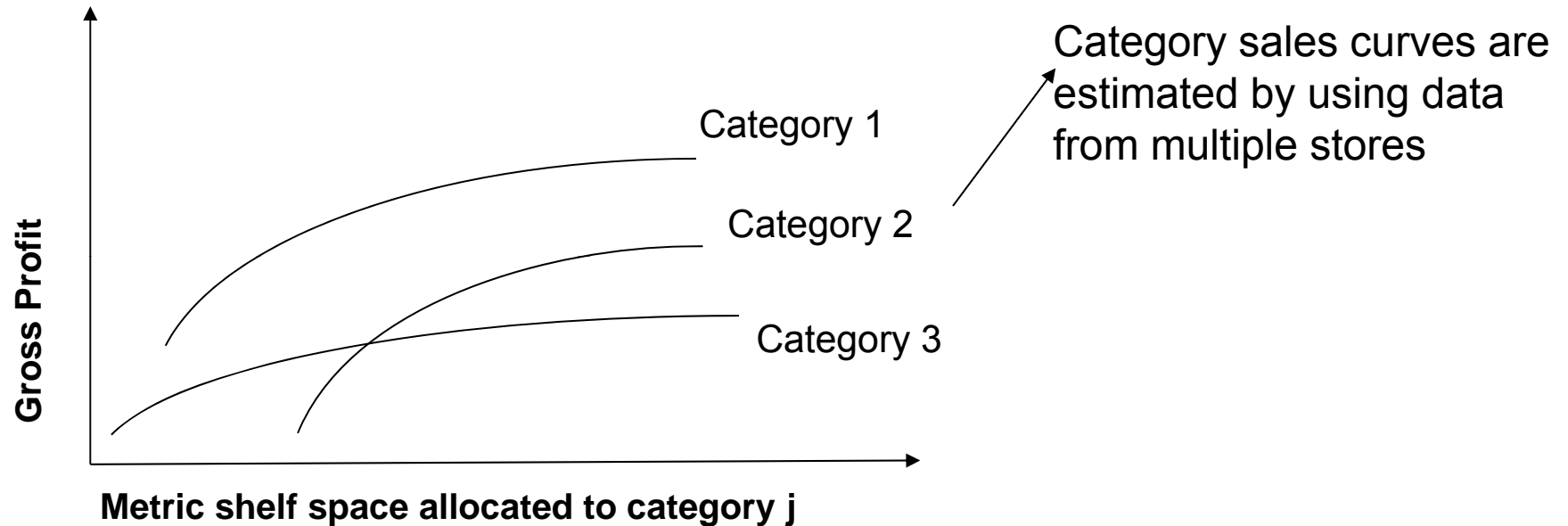
$$RST = \frac{\text{Category Sales}}{\text{No. of Titles}}$$

- Shelf space periodically reassigned from low RST to high RST
- Following the principle of Darwin’s “Natural Selection” and “Survival of the Fittest”, categories “fight” for shelf space
- Store managers allowed to pick titles to be stocked within each category, thereby decentralizing a part of the decision process

# Shelf Space Allocation at Albert Heijn

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Frank Jensen's method



For each store: Allocate meters of space with Greedy method until the store runs out of space

# Assortment Planning at Tanishq

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- No.1 Jewelry retailer in India
- 30,000 SKUs, 15 categories, 52 retail outlets
- Diverse demand patterns across stores
- Assortment planning decisions are
  - Hierarchical at higher attribute levels (category, theme, design)
  - Decentralized for lower attribute levels (models, size), to provide for local customization of stock mix
  - For example, each store to carry 30% of national best sellers, 20% of regional best sellers, 10% of store best sellers, and rest decided at store level

- Best sellers (national, regional, store) are put on automated SKU level replenishment
- Total category inventory, and product attribute mix by store is specified for the rest of the SKUs.
- Store associates modify order quantities for the non best-selling SKUs, while adhering to these specifications

sales  
velocity

High

Low

- Main focus area for store associates.
- Assortment is designed to ensure high degree of variety.

High

% of sales  
by value

Neckwear A

Neckwear B  
Rings B

Rings A

Wedding

Low

- Automated replenishment for the entire category
- Limited role played by store associates
- No focus on individual SKUs
- Assortment selection based on category inventory norms, recommended product attribute mix, and available inventory at the factory

- Guidelines on selling trends provided based on analysis of stores with similar demographics
- Store associates decide on SKUs to include in assortment.
- Order quantities set by store associates to meet the category level inventory norms.

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Localized Assortment Research at Albert Heijn  
Kok and Fisher (2007)

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# Consumer Response to Unavailability of a Product

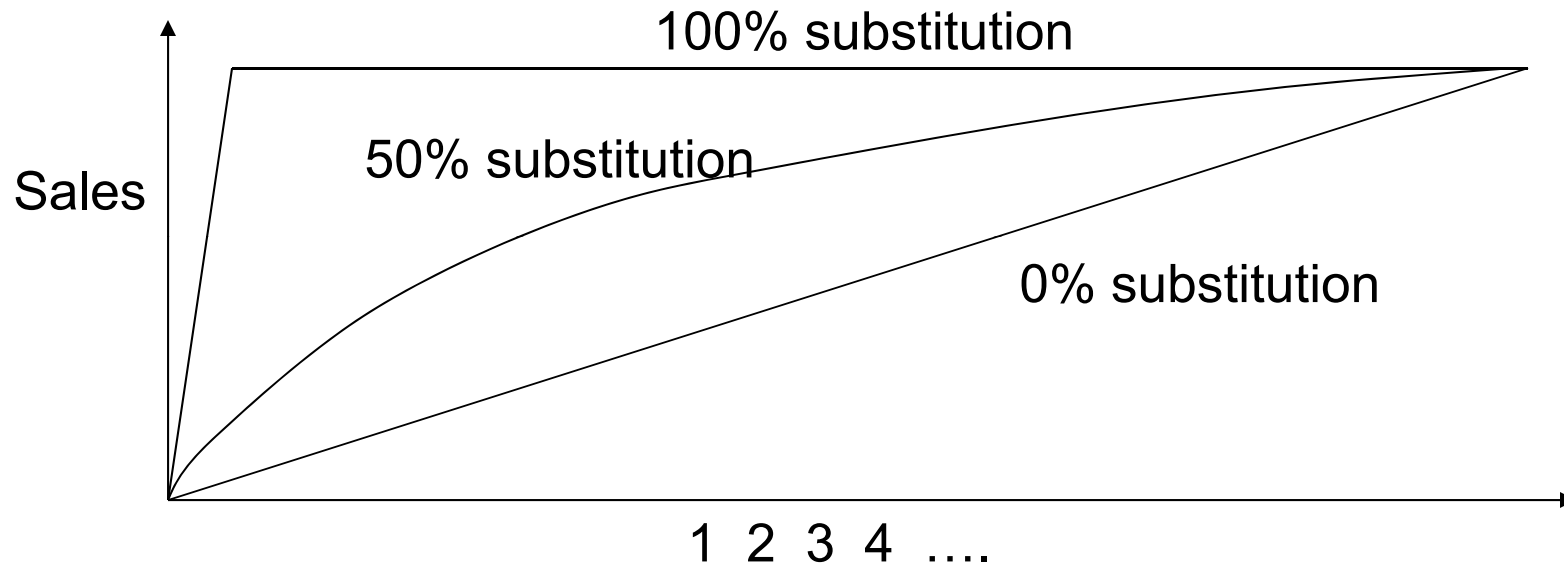
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	Substitute	Delay	Leave
Progressive Grocer (1968)	48%	24%	28%
Walter and Grabner (1975)	83%	3%	14%
Schary and Christopher (1979)	22%	30%	48%
Emmelhainz et al. (1991)	36%	25%	39%
Zinn and Liu (2001)	62%	15%	23%
Gruen et al. (2002)	45%	15%	40%

# Substitution

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Sales response to more choices depends on substitution %



Number of items available to consumer on given day.  
Assume items have equal demand

# Local Assortment Research with Albert Heijn

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$n_j$  = number of facings assigned to SKU  $j$ .

$w_j$  = width of a facing for SKU  $j$

$$\text{Max } \sum_j \text{GP}_j(n_j)$$

Gross profit of  
product  $j$  with facing  
assignment  $n_j$



$$\text{s.t. } \sum_j w_j n_j \leq \text{Available Shelf Space}$$

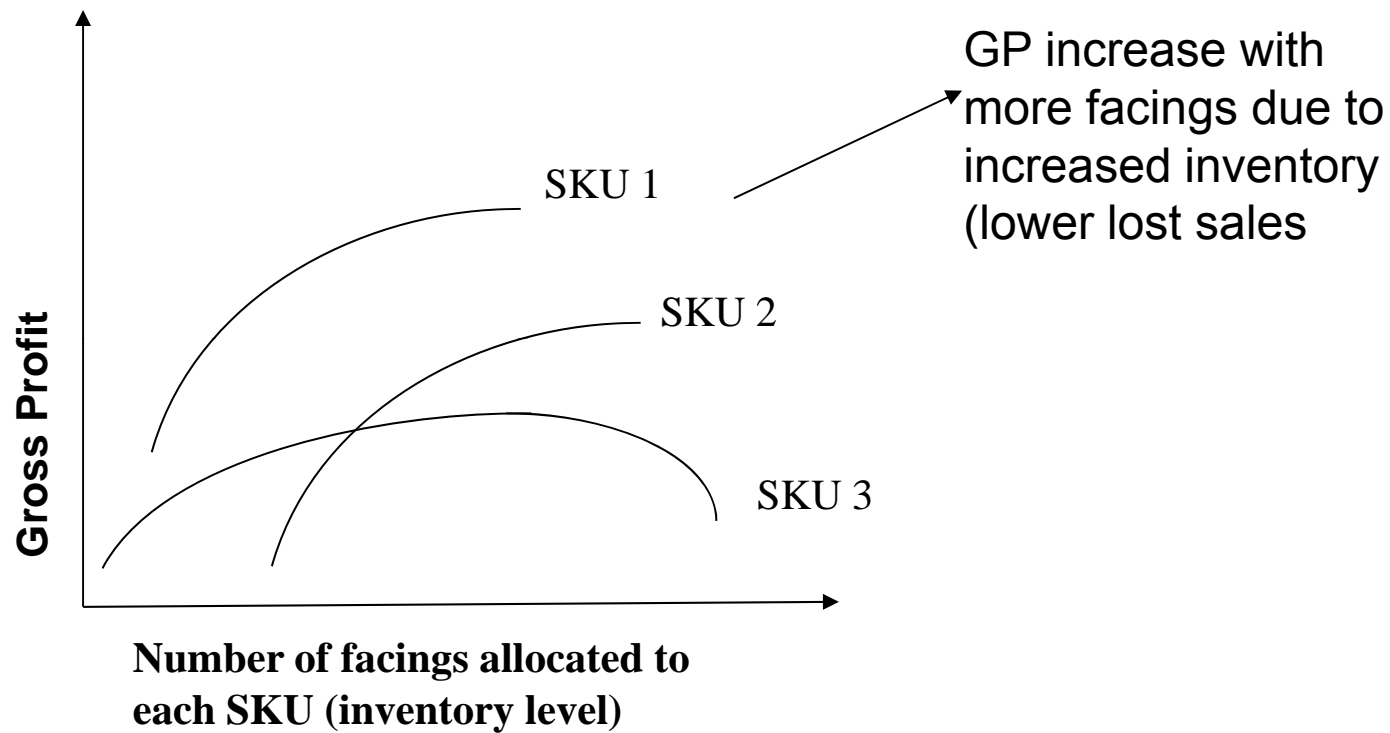
$$n_j \geq 0 \text{ and integral}$$

## Deployment challenges

- Estimating demand for SKUs not currently carried in the store
- Substitution
- Impact of stock outs on product gross profit
- Maximizing  $\sum_j \text{GP}_j(n)$  given interconnectedness – stock out of product A increases demand for B

# Local Assortment Optimization

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**KEY TRADEOFF:**

Breadth (number of items) vs Service Levels (max Inventory Levels)

# Substitution Matrix Examples

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Uniform substitution

$$\begin{bmatrix} 0 & \delta/4 & \delta/4 & \delta/4 \\ \delta/4 & 0 & \delta/4 & \delta/4 \\ \delta/4 & \delta/4 & 0 & \delta/4 \\ \delta/4 & \delta/4 & \delta/4 & 0 \end{bmatrix}$$

Adjacent substitution

$$\begin{bmatrix} 0 & \delta & 0 & 0 \\ \delta/2 & 0 & \delta/2 & 0 \\ 0 & \delta/2 & 0 & \delta/2 \\ 0 & 0 & \delta & 0 \end{bmatrix}$$

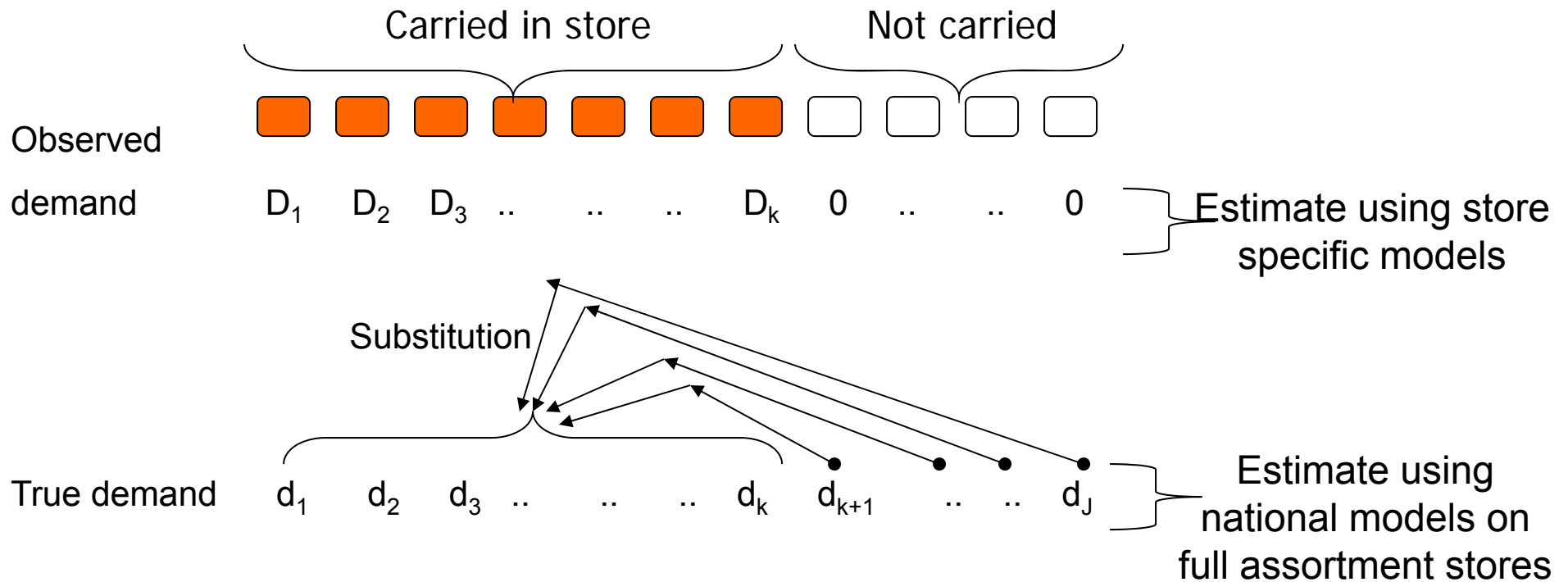
Within subgroups substitution

$$\begin{bmatrix} 0 & \delta & 0 & 0 \\ \delta & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta \\ 0 & 0 & \delta & 0 \end{bmatrix}$$

Proportional substitution

$$\begin{bmatrix} 0 & \delta p_2 & \delta p_3 & \delta p_4 \\ \delta p_1 & 0 & \delta p_3 & \delta p_4 \\ \delta p_1 & \delta p_2 & 0 & \delta p_4 \\ \delta p_1 & \delta p_2 & \delta p_3 & 0 \end{bmatrix}$$

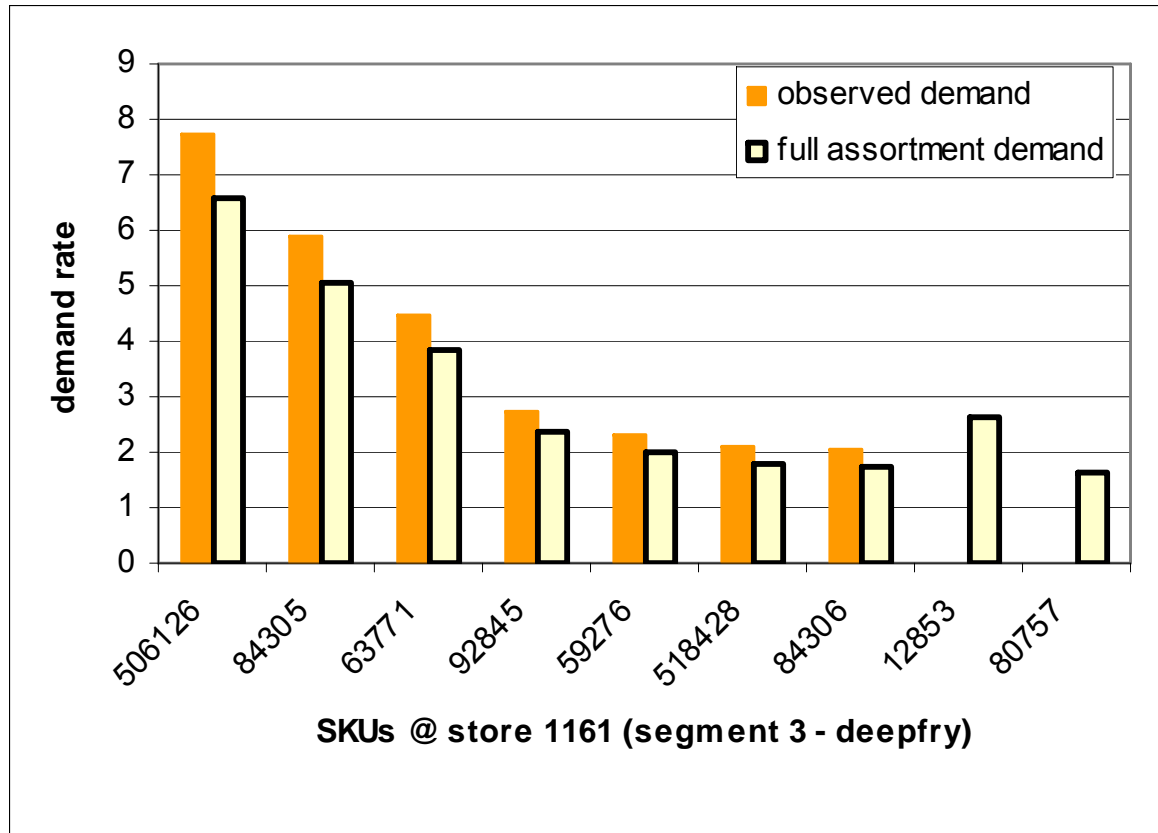
# Estimating Assortment-Based Substitution:



Substitution probability from product  $i$  to  $j$ :

$$\alpha_{ij} = \delta \frac{d_j}{\sum_{l \in N - \{i\}} d_l}$$

# Observed Demand vs. Full Assortment Demand



When a store carries less than full assortment, observed demands are higher than full assortment demand due to substitution

# Estimated Substitution Rates

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segments	Week = 91		Week = 38	
	SR	Error reduction	SR	Error reduction
1	0.3	0%	0	0%
2	0.1	0%	0.3	0%
3	1	1%	1	1%
4	<b>0.9</b>	<b>34%</b>	<b>0.8</b>	<b>30%</b>
5	<b>0.6</b>	<b>38%</b>	<b>0.5</b>	<b>31%</b>
6	<b>0.9</b>	<b>33%</b>	<b>0.7</b>	<b>24%</b>
7	<b>1</b>	<b>20%</b>	<b>1</b>	<b>19%</b>
8	0.1	0%	0	0%
9	0.8	0%	0.8	0%
10	<b>1</b>	<b>29%</b>	<b>1</b>	<b>30%</b>
11	1	5%	1	7%
12	<b>1</b>	<b>36%</b>	<b>1</b>	<b>35%</b>

## Results: Impact on Gross Profit

**Table 6.** Gross profit improvements by our recommended solution and the proportional allocation heuristic over the current assortments at Albert Heijn.

Product category		% Improvement in peak-load gross profit (%)		% Improvement in weekly gross profit (%)	
		Proportional allocation	Recommended solution	Proportional allocation	Recommended solution
3	Average	3.5	13	1.3	7.0
	Min	-3.1	1.6	-2.7	0.7
	Max	45.8	76.0	15.7	30.9
55	Average	3.6	15	0.3	5.6
	Min	-5.0	0.0	-3.4	0.3
	Max	30.3	65.4	8.8	19.3
Grand average		3.6	13.9	0.7	6.2

## Impact of Substitution

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**Table 8.** Impact of the substitution rate on assortment size, shelf space, and total number of facings in a subcategory.

	Substitution rate (%)		
	Low	Medium	High
Average magnitude of assortment size increase	77	29	28
Average magnitude of assortment size decrease	-24	-26	-29
Average change in shelf space	12	-3	-3
Average change in number of facings	11	-3	-2

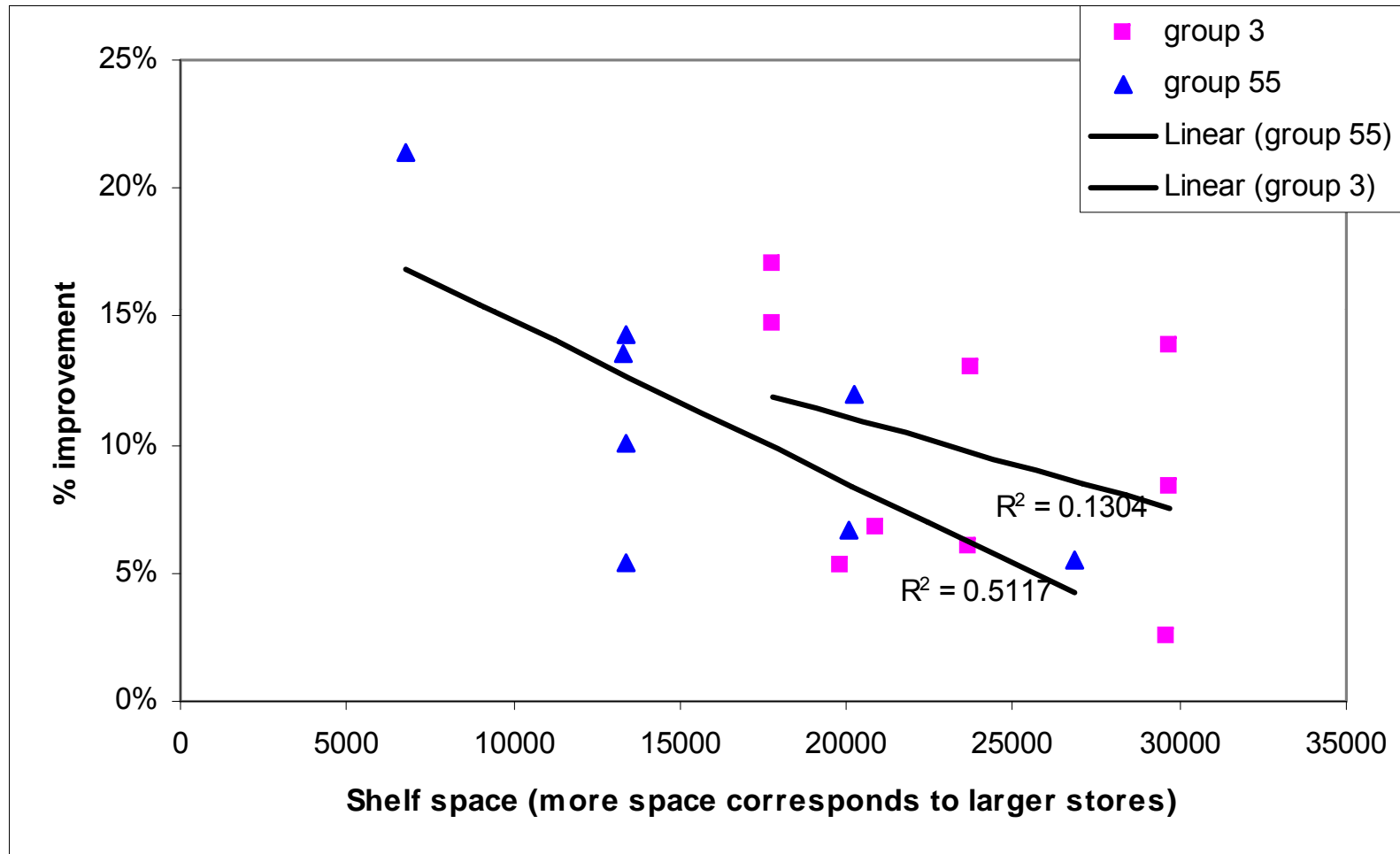
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## Sources of Improvement: Example

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Recommended changes			
# SKUs	Space (%)	GP (%)	Other
-2	6	67	Added 1 facing to 4 products.
-1	-19	76	Added 2 facings to a product with a very large case size.
0	26	54	Reduced facings from a product with decreasing GP (due to disposals) and increased facings for the others by 1.
-1	-15	6	Dropped a product with negative GP, increased 2 facings for 2 products.
-1	-25	50	Dropped a low-margin product and added 3 facings to a product with higher margin and large case size.
0	48	63	Added facings to 4 products and reduced 1 facing from a product.
-3	-78	-18	Dropped products require 3–4 facings for significant sales because case sizes are twice the capacity of facings, but demand and margins are low.
-5	-26	181	Dropped 4 products with very low GP; added facings to others, all large case size, one also with high margin.
0	79	195	Added 8 facings to 2 products with case sizes 1.5 times facing capacity and 2 facings to 2 products with also large cases.

# More Improvement When Shelf Space is Tight



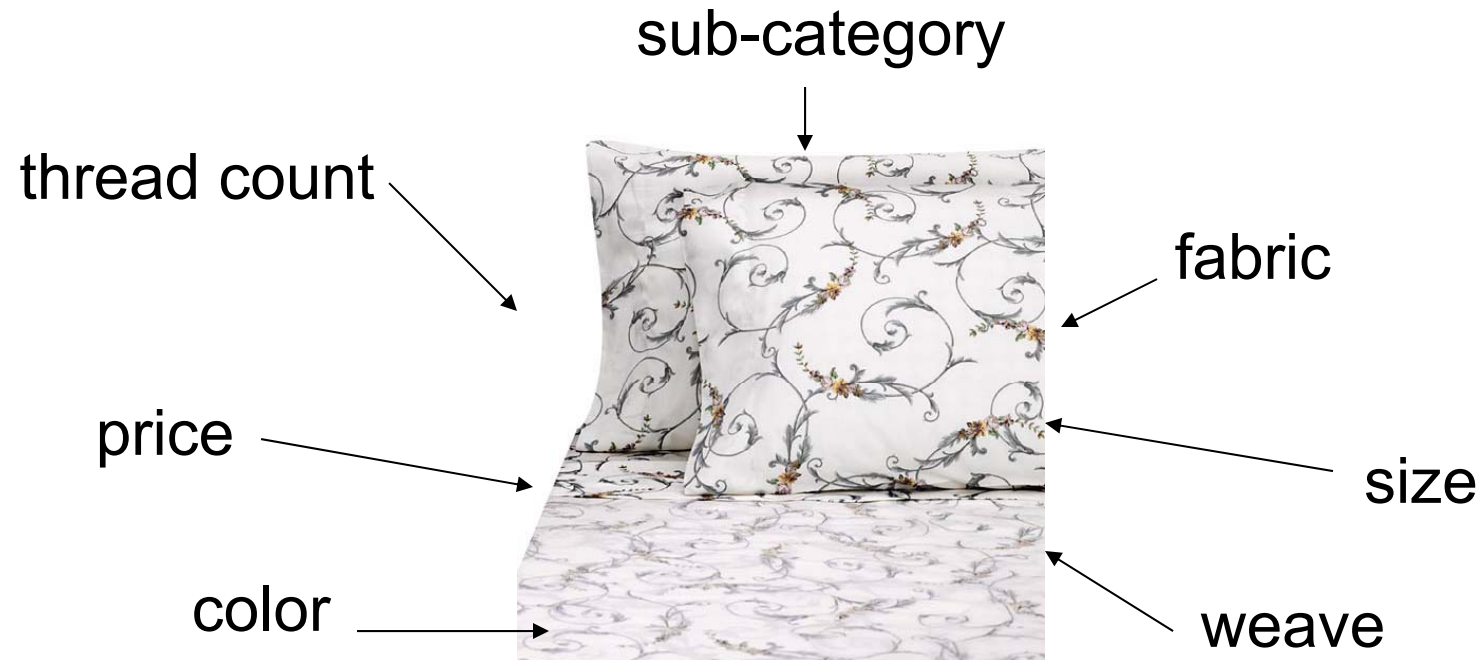
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Attribute-Based Substitution Estimation  
Vaidyanathan and Fisher (2007)

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# Identify Relevant Product Attributes

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- Attribute-based view is useful for
  - Understanding substitution behavior
  - Choosing localization objectives
  - Estimating demand for new products (Fader and Hardie 1996)

# Snack Cakes:

## Estimate Sales Potential of all SKUs and Substitution Rates

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Flavor	Single Serve		Family Size	
	Brand 1	Brand 2	Brand 1	Brand 2
Chocolate	5,986	43	570	161
Cinnamon	4,345	1,414	188	458
Peanut Butter	4,358		579	
Butterscotch	3,422		501	
Fudge	3,247	418		
Choc. Chip	2,613		892	
Oatmeal/Raisin	2,942			
Honey	1,662	702		
Butter	961	1,173		77
Raspberry		1,717		303
Cheese	625	1,037		319
Vanilla	1,300			66
Coconut	1,309			
Cherry/Cheese		1,209		
Pineapple/Cheese		1,021		
Vanilla/Chocolate		840		

# Snack Cakes: Estimated Substitution Rates

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Substitution Probability Estimates

	$B_1S$	$B_2S$	$B_1F$	$B_2F$
$B_1S$	1	18%	0	0
$B_2S$	26%	1	0	0
$B_1F$	0	0	1	89%
$B_2F$	0	0	22%	1

Snack Cakes: Demand Share Estimates (Averaged across

Brand Size	Sales Share	Demand Share
$B_1S$	67%	61%
$B_2S$	24%	27%
$B_1F$	4%	6%
$B_2F$	5%	6%

## Assortment Changes for one Store

Flavor	Single Serve		Family Size		Demand Rank	Brand Sizes
	Brand 1	Brand 2	Brand 1	Brand 2		
Cinnamon	-	-	x	-	1	3
Chocolate	-	-	x	-	2	3
Peanut Butter	-	+	x	+	3	3
Butterscotch	-	+	x	+	4	3
Butter	-	-		-	5	3
Vanilla	-	+		-	6	3
Raspberry	+	-		-	7	3
Fudge	-	-		+	8	3
Honey	-	-		+	9	3
Buttercream	+	+	x	+	10	3
Choc. Chip	-		x	+	11	2
Cherry/Cheese	+	x		+	12	2

# Tires: Sales Data from one Store

Size	National High	National Medium	House 1 High	House 2 High	House 2 Medium	House Low
P205/65R15	72	64	20	272	570	
P215/70R15	30	21		334	340	
P225/60R16	56		97	285	763	
P215/60R16	10		16	70	76	
P225/70R15			8	100	73	
P185/70R14			8	95	223	
P195/65R15				152	298	
P215/65R15				144	221	
P175/80R13					5	20
P205/75R14					10	299
P195/70R14		7	33	157	377	
P215/65R16				221		
P185/65R15				135		
175/70R13						236
185/65R14						114
P235/60R16			61			
P185/60R14					101	
P196/60R14					115	
P205/75R15	8					200
P205/70R15		10		272	524	
P185/65R14		30		225	568	
P235/75R15	12				5	167

## Tires: Substitution Estimates

Table 9: Management's estimate of the most likely substitution probabilities

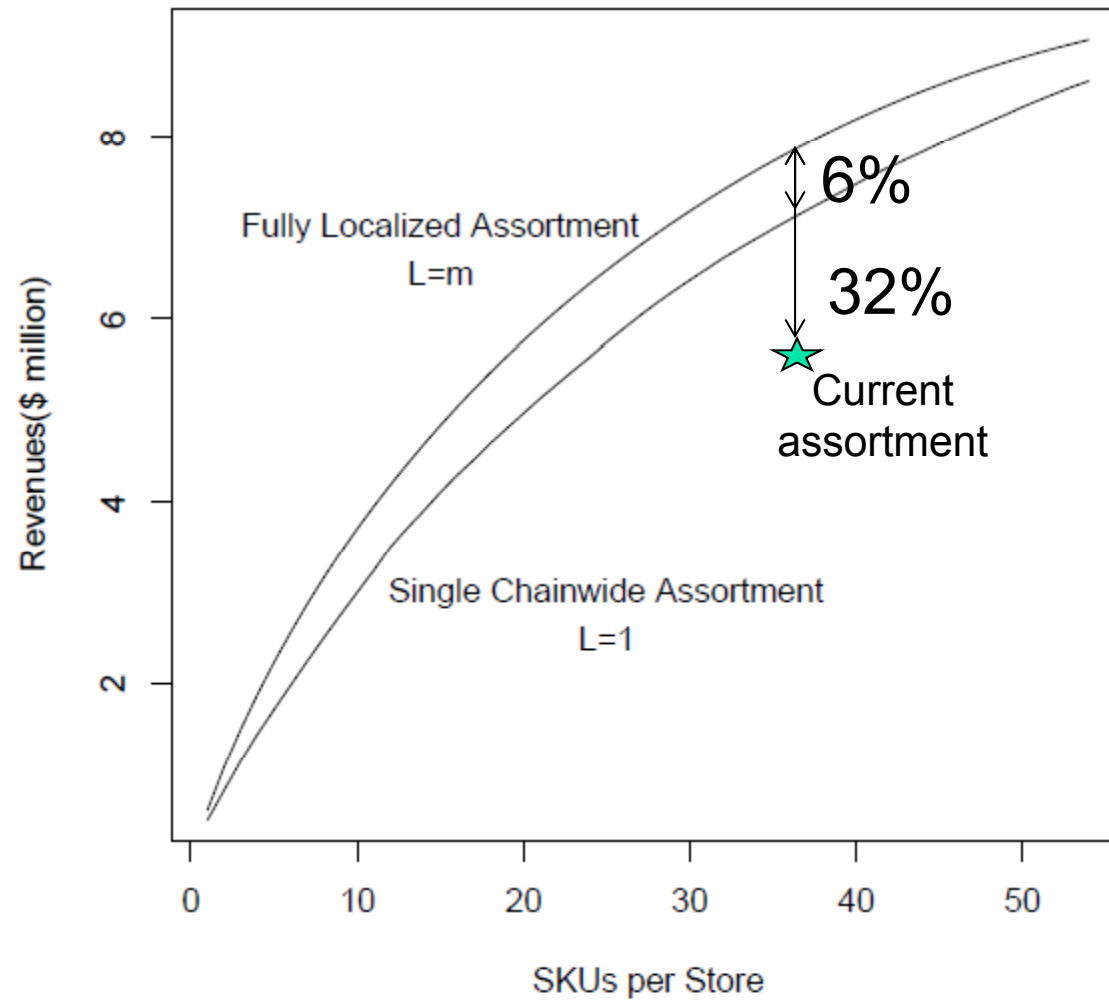
	<i>NH</i>	<i>NM</i>	<i>H1H</i>	<i>H2H</i>	<i>H2M</i>	<i>H3L</i>
<i>NH</i>	1	<i>S</i>	<i>S</i>	<i>S</i>	0	0
<i>NM</i>	<i>L</i>	1	<i>S</i>	<i>S</i>	0	0
<i>H1H</i>	0	0	1	<i>L</i>	<i>S</i>	0
<i>H2H</i>	0	0	<i>S</i>	1	<i>S</i>	0
<i>H2M</i>	0	0	<i>S</i>	<i>L</i>	1	0
<i>H3L</i>	0	0	0	0	<i>M</i>	0

*S = Somewhat Likely, L = Likely, M = Most Likely*

Tires: Substitution Probability Estimates (Averaged)

Substitution Likelihood	Probability
Somewhat Likely	2%
Likely	6%
Most Likely	45%

# Impact of Localization



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# Hierarchical Choice Models

## Kok and Xu (2011)

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# Subgroups Based on Product Type

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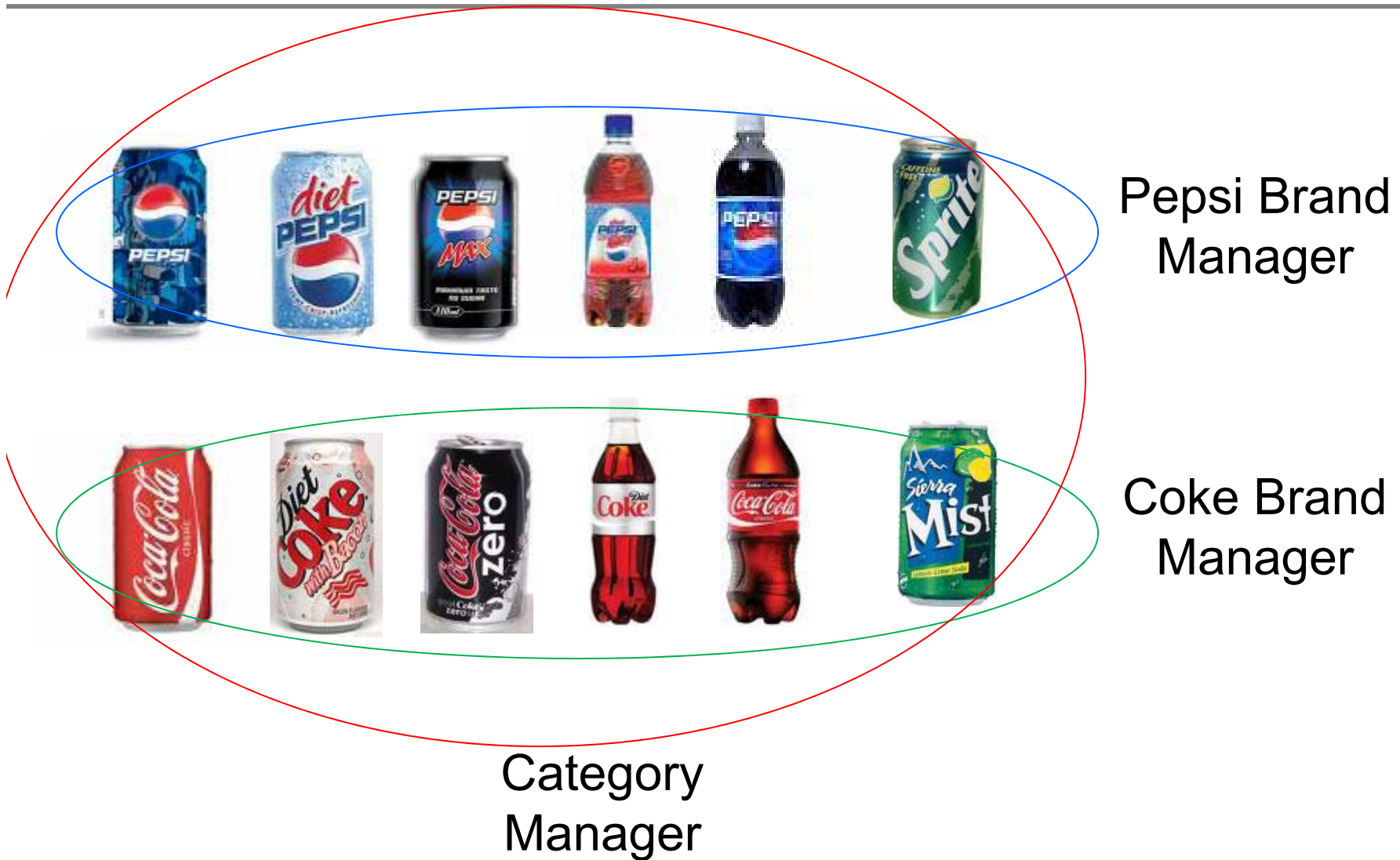


## Subgroups Based on Brand

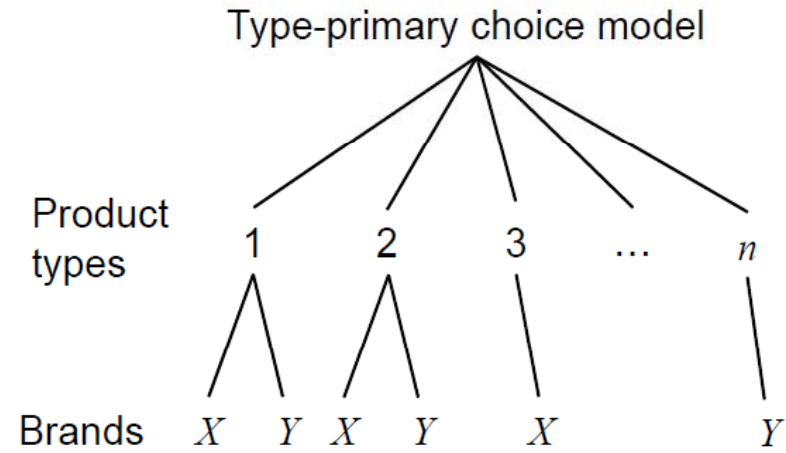
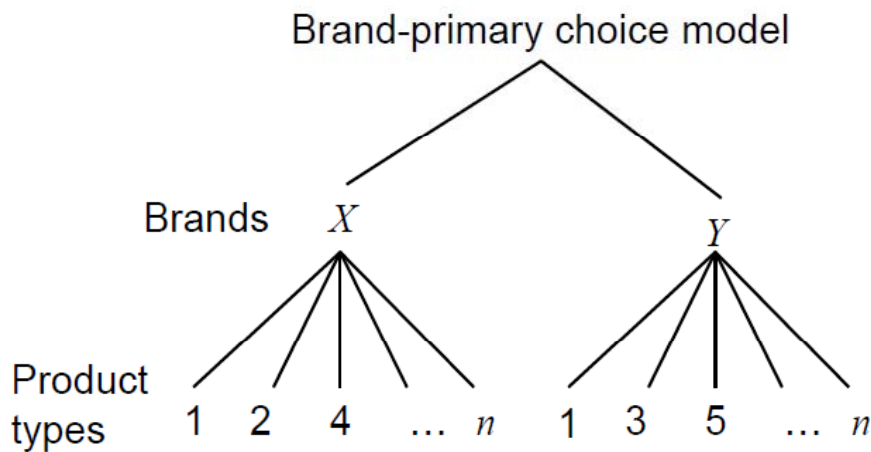
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# Assortment Planning and Pricing Regimes



# Brand-Primary Model vs. Type-Primary Model



- Product types are not functionally differentiated or brand loyalty is strong
  - Dress shirts with different colors
  - Ice creams with different flavors
  - Detergents with different scents

- Product types are functionally differentiated
  - Regular vs. diet drinks,
  - Regular vs. decaf coffee,
  - Sedan vs. mini-van,
  - Point-and-shoot vs. SLR camera

(Kannan and Wright 1991, Urban et al. 1984, Grover and Dillon 1985, Allenby 1989)

# Type-Primary Model: Centralized Management

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	Product types						
	1	2	3	4	5	6	7
Brand X	X	X		X			
Brand Y	Y	Y	Y		Y		

Can be optimal

	1	2	3	4	5	6	7
Brand X	X	X		X	X		
Brand Y	Y	Y	Y		Y		Y

Cannot be optimal

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Dynamic Assortment Customization  
in Online Retailing  
Bernstein, Kok and Xie (2011)

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# Example: Online Retailer

The screenshot displays the Amazon.com homepage with a search for "basketball shoes". The navigation bar includes the Amazon logo, a personalized greeting for "Gurhan Kok" (circled in red), and links for "Today's Deals", "Gifts & Wish Lists", and "Gift Cards". A promotional banner for "wishlistSweepstakes Week 1" is visible. The search bar shows "Shoes" as the selected category and "basketball shoes" as the search term. Below the search bar, a navigation menu lists various shoe categories: Women's, Men's, Juniors', Girls', Boys', Athletic & Outdoor, Designer, Handbags, Accessories, Apparel, and Sellers. The main content area shows search results for "basketball shoes" with 3,445 results. The results are sorted by Relevance and displayed in a grid. The first three items are:

- 1. Reebok Men's ATR Lock It Up Full Hex Ride Basketball Shoe: Price range \$36.38 - \$89.99, 10 reviews.
- 2. adidas Men's Lift Off II Basketball Shoe: Price range \$30.24 - \$66.90, 1 review.
- 3. Reebok Men's RBK Flash Hexalite Basketball Shoe: Price range \$34.01 - \$67.99, 4 reviews.

The left sidebar contains filters for Department (Shoes), Color (Any Color), Shipping Option (Prime Eligible), and Brand (adidas).

# Example: Online Retailer

amazon.com Hello, Gurhan Kok. We have [recommendations](#) for you. (Not Gurhan?)  
Gurhan's Amazon.com | Today's Deals | Gifts & Wish Lists | Gift Cards

Shop All Departments Search Shoes

Shoes Women's Men's Juniors' Girls' Boys' Athletic & Outdoor Designer Handbags

adidas Men's Lift Off II Basketball Shoe  
Other products by [adidas](#)  
★★★★☆ (1 customer review) | [More about this product](#)

List Price: ~~\$66.95~~  
Price: **\$35.58** & this item ships for **FREE with Super Saver Shipp**  
You Save: **\$31.37 (47%)**

**To Buy, Add to Shopping Cart**

Size: **8 D(M) US**

6.5 D(M) US	7 D(M) US	7.5 D(M) US	<b>8 D(M) US</b>	9 D(M) US	9.5 D(M) L
10 D(M) US	10.5 D(M) US	11 D(M) US	11.5 D(M) US	12 D(M) US	13
14 D(M) US	18 D(M) US				

Color: **White/White/Indigo**

**ZOOM**  
[See larger image and other views \(with zoom\)](#)

# Example: Online Retailer

amazon.com Hello, Gurhan Kok. We have [recommendations](#) for you. ([Not Gurhan?](#))  
Gurhan's Amazon.com | [Today's Deals](#) | [Gifts & Wish Lists](#) | [Gift Cards](#)

Shop All Departments Search Shoes

Shoes Women's Men's Juniors' Girls' Boys' Athletic & Outdoor Designer Handbags

adidas Men's Lift Off II Basketball Shoe  
Other products by [adidas](#)  
★★★★☆ (1 customer review) | [More about this product](#)

Price: \$30.24 - \$66.99  
Sale: \$49.99 on selected colors & sizes  
[Special Offers Available](#)

**Sorry, this item is not available in  
Size: 8 D(M) US  
Color: White/White/Royal**

Select Color

Size: 8 D(M) US

6.5 D(M) US	7 D(M) US	7.5 D(M) US	8 D(M) US	9 D(M) US	9.5 D(M) US
10 D(M) US	10.5 D(M) US	11 D(M) US	11.5 D(M) US	12 D(M) US	13
14 D(M) US	18 D(M) US				

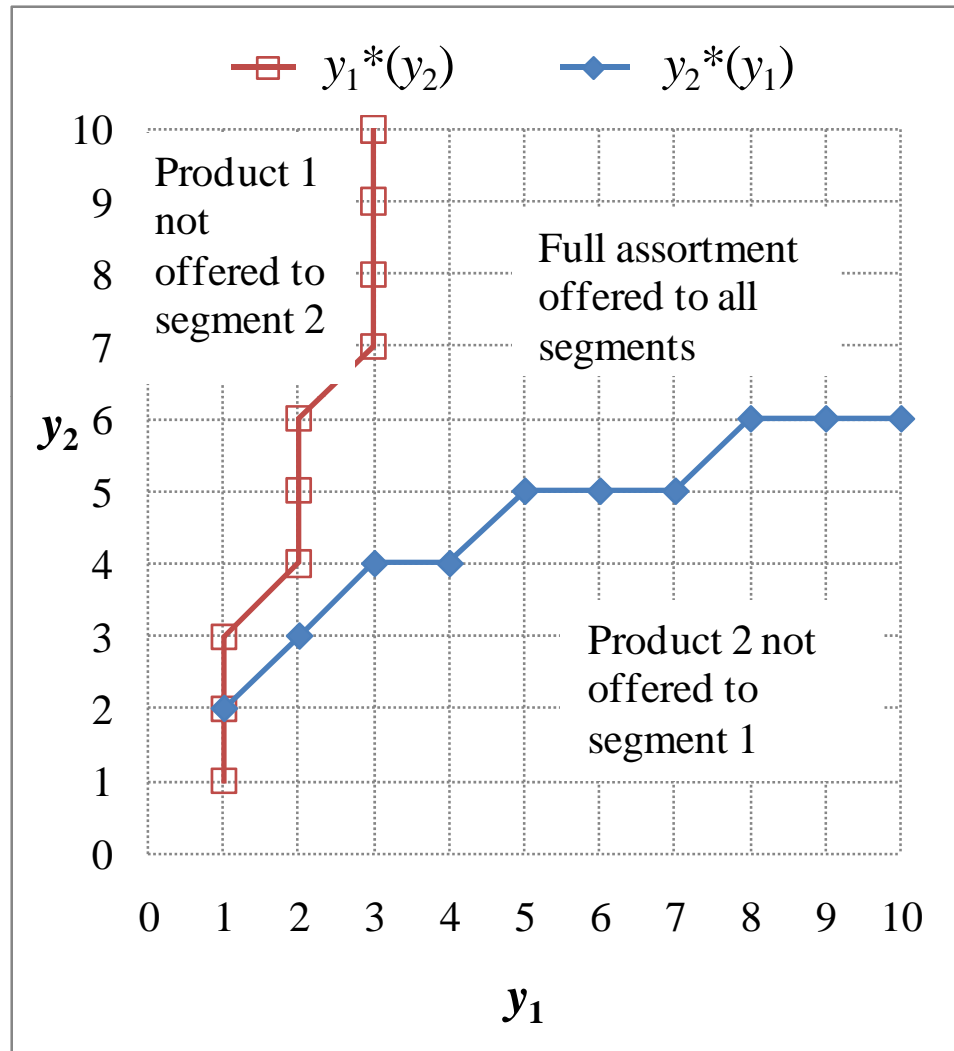
Color: White/White/Royal

See larger image and other views (with zoom)

Zoom

Color selection: [Black] [White] [White/Royal] [White/White/Royal]

# Characterizing the Policy



# Broader Questions on Customization

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## Customer behavior

- Impact of sort order on purchasing behavior
- Can we refine estimates based on clicks during a search?

## Revenue maximization and demand management

- Customization based on customer characteristics
- Customization based on margins
- Customization based on inventory levels or supply outlook

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# Thank you!

Papers are available at my web site.

<http://faculty.fuqua.duke.edu/~agkok/>

Please send comments and stories to

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